Comparatively tougher chapter to get marks..

INDEFINITE & DEFINITE INTEGRATION (II-B)

In intermediate public exams 3 LAQs, 3 or 4 VSAQs are expected from this chapter. In JEE-Mains, Advanced one or two questions are compulsory. In EAMCET 6-7 questions are expected from this chapter. It is comparatively tougher chapter to get marks, so much practice is needed.

EAMCET Previous Questions

1. $\int_0^{\pi/4}\left[\sqrt{\tan x}+\sqrt{\cot x}\right]dx=$

(TS EAMCET-2016)

1) $\frac{\pi}{\sqrt2}$ 2) $\frac{\pi}{2}$ 3) $\frac{3\pi}{\sqrt2}$ 4) π

Hint: $\int_0^{\pi/4}\frac{\sin x+\cos x}{\sqrt{\sin x\cos x}}dx$

put $\sin x-\cos x=t$

$\Rightarrow(\cos x+\sin x)dx=dt$

Upper limit = 0, lower limit = –1 & and squaring gives $\sin 2x=1-t^2$

$\int_{-1}^{0}\frac{1}{\sqrt{\frac{1-t^2}{2}}}dt=\sqrt2\left[\sin^{-1}t\right]_{-1}^{0}=\frac{\pi}{\sqrt2}$

Ans: 1

2. $\int_0^{\pi/4}\frac{\sin x+\cos x}{7+9\sin 2x}dx=$

(TS EAMCET-2016)

1) $\frac{\log 3}{4}$ 2) $\frac{\log 3}{36}$

3) $\frac{\log 7}{12}$ 4) $\frac{\log 7}{24}$

Hint: $I=\int_0^{\pi/4}\frac{\sin x+\cos x}{7+9\sin 2x}dx$

put $\sin x-\cos x=t$

$\Rightarrow(\cos x+\sin x)\,dx=dt$

Upper limit = 0, Lower limit = –1

As $\sin x-\cos x=t$ squaring gives $\sin 2x=1-t^2$

$I=\int_{-1}^{0}\frac{dt}{7+9\left(1-t^2\right)}=\frac{1}{24}\log\left[\frac{4+3t}{4-3t}\right]_{-1}^{0}$

$=\frac{\log 7}{24}$

Ans: 4

3. By the definition of the definite integral, the value of

$\lim_{x\to\infty}\left(\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-2^2}}+..+\frac{1}{\sqrt{n^2-(n-1)^2}}\right)$

is equal to: **(AP EAMCET-2016)**

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

Hint: Using limit of summation

formula $\lim_{n\to\infty}\sum_{1}^{n-1}\frac{1}{\sqrt{n^2-r^2}}=$

$\int_0^1\frac{1}{\sqrt{1-x^2}}dx=\frac{\pi}{2}$

Ans: 2

4. If $\int x^3e^{5x}dx=\frac{e^{5x}}{5^4}\left(f(x)\right)+c$, then $f(x)=$

(AP EAMCET-2016)

1) $\frac{x^3}{5}-\frac{3x^2}{5^2}+\frac{6x}{5^3}-\frac{6}{5^4}$

2) $5x^3-5^2x^2+5^3x-6$

3) $5^2x^3-15x^2+30x-6$

4) $5^3x^3-75x^2+30x-6$

Hint: $\int x^3e^{5x}dx=$

$x^3\frac{e^{5x}}{5}-3x^2\frac{e^{5x}}{25}+6x\frac{e^{5x}}{125}-\frac{6e^{5x}}{625}+c$

$=\frac{e^{5x}}{625}\left[125x^3-75x^2+30x-6\right]+c$

Gives option 1 *Ans: 1*

5. $\int_0^{\pi/2}\frac{16x\sin x\cos x\,dx}{\sin^4x+\cos^4x}=$

(TS EAMCET-2015)

1) $\frac{\pi^2}{4}$ 2) $\frac{\pi^2}{2}$ 3) π^2 4) $2\pi^2$

Hint: $I=\int_0^{\pi/2}\frac{16x\sin x\cos x}{\sin^4x+\cos^4x}dx$

apply $\int_0^a f(a-x)=\int_0^a f(x)$ property

$I=\int_0^{\frac{\pi}{2}}\frac{16\left(\frac{\pi}{2}-x\right)\cos x\sin x}{\cos^4x+\sin^4x}dx$

$I=16\left(\frac{\pi}{2}\right)\int_0^{\pi/2}\frac{\sin x\times\cos x}{\sin^4x+\cos^4x}dx-I$

$2I=16\left(\frac{\pi}{2}\right)\int_0^{\pi/2}\frac{\sin x\times\cos x}{\sin^4x+\cos^4x}dx$

$I=4\pi\int_0^{\pi/2}\frac{\tan x\sec^2x}{1+\tan^4x}dx$

$=2\pi\int_0^{\pi/2}d[\tan^{-1}(\tan^2x)]=2\pi\left(\frac{\pi}{2}-0\right)=\pi^2$

Ans: 3

6. $\int_0^1\sqrt{\frac{1-x}{1+x}}dx=$ **(TS EAMCET-2015)**

1) $\frac{\pi}{2}-1$ 2) $\frac{\pi}{2}+1$

3) $\pi-1$ 4) $\frac{3\pi}{2}$

Hint: $\int_0^1\frac{1-x}{\sqrt{1-x^2}}dx$

$=\left[\sin^{-1}x\right]_0^1+\left[\sqrt{1-x^2}\right]_0^1=\frac{\pi}{2}-1$

Ans: 1

7. $\int\sqrt{e^x-4}\,dx=$ *(TS EAMCET-2015)*

1) $\tan^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right)+\sqrt{e^x-4}+c$

2) $2\sqrt{e^x-4}-4\tan^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right)+c$

3) $2\sqrt{e^x-4}-4\cot^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right)+c$

4) $\sqrt{e^x-4}-4\tan^{-1}\left(\sqrt{e^x-4}\right)+c$

Hint: let $e^x-4=t^2$

$e^xdx=2tdt\Rightarrow dx=\frac{2tdt}{t^2+4}$

then $I=\int\frac{2t^2dt}{t^2+4}=2\int\left(1-\frac{4}{t^2+4}\right)dt$

$=2\sqrt{e^x-4}-4\tan^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right)+c$

Ans: 2

8. $\int e^x\frac{x^2+1}{(x+1)^2}dx=$

(AP EAMCET-2015)

1) $\frac{e^x}{x+1}+c$ 2) $\frac{-e^x}{x-1}+c$

3) $e^x\left(\frac{x-1}{x+1}\right)+c$ 4) $e^x\left(\frac{x+1}{x-1}\right)+c$

Hint: $\int e^x\frac{x^2+1}{(x+1)^2}dx=\int e^x\frac{x^2-1+2}{(x+1)^2}dx=$

$\int e^x\left(\frac{x-1}{(x+1)}+\frac{2}{(x+1)^2}\right)dx=e^x.\frac{x-1}{x+1}$

Ans: 3

Important problems (IPE)

1. Evaluate $\int\frac{1}{(1+x)\sqrt{3+2x-x^2}}dx$

Sol: Put $\boxed{1+x=\frac{1}{t}}$

$x=\frac{1}{t}-1\Rightarrow dx=\frac{-1}{t^2}dt$

Consider $3+2x-x^2$

$=3+2\left(\frac{1}{t}-1\right)-\left(\frac{1}{t}-1\right)^2$

$=3+2\frac{1}{t}-2-\left[\frac{1}{t^2}+1-\frac{2}{t}\right]$

$=3+\frac{2}{t}-2-\frac{1}{t^2}-1+\frac{2}{t}$

$=\frac{4}{t}-\frac{1}{t^2}=\frac{4t-1}{t^2}$

$\therefore\int\frac{1}{(1+x)\sqrt{3+2x-x^2}}dx=$

$\int\frac{1}{\frac{1}{t}\sqrt{\frac{4t-1}{t^2}}}\left(\frac{-1}{t^2}\right)dt$

$=\int\frac{1}{\frac{1}{\not t}\frac{\sqrt{4t-1}}{\not t^2}}\left(\frac{-1}{\not t^2}\right)dt$

$=-\int\frac{1}{\sqrt{4t-1}}dt=\frac{-\not2\sqrt{4t-1}}{\not4_2}+c$

$=\frac{-1}{2}\sqrt{4\left(\frac{1}{1+x}\right)-1}+c$

$=\frac{-1}{2}\sqrt{\frac{4-(1+x)}{1+x}}+c$

$=\frac{-1}{2}\sqrt{\frac{3-x}{1+x}}+c$

2. If $I_n=\int\sin^nx.dx$ then prove that $I_n=\frac{-\sin^{n-1}x.\cos x}{n}+\frac{n-1}{n}I_{n-2}$

Sol: Let $I_n=\int\sin^nx.dx$ ....(1)

$I_n=\int\sin^{-1}x\sin x.dx.$ Use product rule,

$I_n=\sin^{n-1}x\int\sin x.dx-$

$\int\left[\frac{d}{dx}\left(\sin^{n-1}x\right)\int\sin x dx\;dx\right]$

$I_n=\sin^{n-1}x\left(-\cos x\right)-$

$\int(n-1)\sin^{n-1-1}x.\cos x\left(-\cos x\right)dx$

$I_n=-\sin^{n-1}x.\cos x+$

$(n-1)\int\sin^{n-2}x.\cos^2x\,dx$

$=-\sin^{n-1}x.\cos x+$

$(n-1)\int\sin^{n-2}x\left(1-\sin^2x\right)dx$

$=-\sin^{n-1}x\cos x+(n-1)$

$\int\sin^{n-2}x.dx-(n-1)\int\sin^nx.dx$

$I_n=-\sin^{n-1}x\cos x+(n-1)$

$I_{n-2}-(n-1)I_n$ [from (1)]

$I_n+(n-1)I_n=-\sin^{n-1}x.\cos x$ $+(n-1)I_{n-2}$

$I_n+(\not1+n-\not1)$

$=-\sin^{n-1}x.\cos x+(n-1)I_{n-2}$

$\boxed{I_n=\frac{-\sin^{n-1}x.\cos x}{n}+\frac{n-1}{n}.I_{n-2}}$

3. Evaluate $\int_0^1\frac{\log(1+x)}{1+x^2}.dx$

Sol: Let $I=\int_0^1\frac{\log(1+x)}{1+x^2}.dx$....(1)

put $x=\tan\theta\Rightarrow dx=\sec^2\theta.d\theta$

If $x=1\Rightarrow1=\tan\theta\Rightarrow\theta=\pi/4$

If $x=0\Rightarrow0=\tan\theta\Rightarrow\theta=0$

$I=\int_0^{\pi/4}\frac{\log(1+\tan\theta)}{1+\tan^2\theta}.\sec^2\theta.d\theta$

$I=\int_0^{\pi/4}\frac{\log(1+\tan\theta)}{\not\sec^2\theta}.\not\sec^2\theta.d\theta$

$I=\int_0^{\pi/4}\log(1+\tan\theta)d\theta$ ......(2)

as $\boxed{\int_0^af(x).dx=\int_0^af(a-x)dx}$

$I=\int_0^{\pi/4}\log\left[1+\tan\left(\pi/4-\theta\right)\right].d\theta$

$I=\int_0^{\pi/4}\log\left[1+\frac{\tan\pi/4-\tan\theta}{1+\tan\pi/4\tan\theta}\right]d\theta$

$I=\int_0^{\pi/4}\log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right].d\theta$

$I=\int_0^{\pi/4}\log\left[\frac{1+\not\tan\theta+1-\not\tan\theta}{1+\tan\theta}\right].d\theta$

$I=\int_0^{\pi/4}\log\left[\frac{2}{1+\tan\theta}\right].d\theta$

$I=\int_0^{\pi/4}\log2\;d\theta-\int_0^{\pi/4}\log\left(1+\tan\theta\right)d\theta$

$I=\log2\int_0^{\pi/4}1.\,d\theta-I$ [*from*(2)]

$I+I=\log2\int_0^{\pi/4}1.\,d\theta$

$2I=\log2\left[\theta\right]_0^{\pi/4}=\log2\left[\pi/4-0\right]$

$=(\log2).\pi/4$ $\boxed{\therefore I=\pi/8.\log2}$

4. Evaluate $\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin2x}dx$

put $\sin2x=1-(\sin x-\cos x)^2$

$=\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\left[1-(\sin x-\cos x)^2\right]}.dx$

Put $\sin x-\cos x=t$

$\Rightarrow(\cos x+\sin x)dx=dt$

If $x=\pi/4\Rightarrow t=0$

If $x=0\Rightarrow t=-1$

$\therefore\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin2x}dx=$

$\int_{-1}^{0}\frac{1}{9+16\left(1-t^2\right)}.dt$

$=\int_{-1}^{0}\frac{1}{25-16t^2}.dt$

$=\int_{-1}^{0}\frac{1}{(5)^2-(4t)^2}.dt$

$=\frac{1}{4\times2\times5}\log\left|\frac{5+4t}{5-4t}\right|_{-1}^{0}$

$\left[\because\int\frac{1}{a^2-x^2}dx=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|\right]$

$=\frac{1}{40}\left[\log\left|\frac{5-0}{5-0}\right|-\log\left|\frac{5+4(-1)}{5-4(-1)}\right|\right]$

$=\frac{1}{40}\left[\log|1|-\log\left|\frac{1}{9}\right|\right]$

$=\frac{1}{40}\left[0-\log|3|^{-2}\right]$

$=\frac{-1}{40}(-2).\log3$

$=\frac{1}{20}\log3$